Roll No. ..... Total No. of Pages : 4

BT-2/J08 8246

Mathematics—II (2004 onwards)

Paper : Math-102E

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt **FIVE** questions in all, taking at least **ONE** question from each unit.

UNIT—I

1. (a) Define equivalent matrices, and find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

(b) Find the values of 'a' and 'b' for which the equations $x + 5y + 3z = 9$; $x + ay + z = 3$ and $x + 2y + 2z = b$ are consistent. When will these equations have unique solution ? 10,10

2. (a) Verify Cayley Hamilton theorem for the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

and find its inverse.

(b) Reduce the quadratic form; $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to a canonical form and find the matrix of transformation. 10,10

UNIT—II

3. (a) Solve

$$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0.$$

(b) Find the orthogonal trajectories of;

$$r = a(1 + \cos\theta).$$

(c) Find the complete solution of;

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x.$$ 6,6,8

4. (a) Solve by the method of undetermined coefficients;

$$(D^2 - 3D + 2)y = e^x + x^2.$$ 8

(b) An alternating e.m.f. E sin pt is applied to a circuit at t = 0. Given the equation for the current 'i' as

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = pE\cos pt,$$

find the current 'i' when (i) $CR^2 > 4L$, (ii) $CR^2 < 4L$. 12

UNIT—III

5. (a) Define Laplace transform and prove that,

$$L[f'(t)] = s\bar{f}(s) - f(o)$$

where $\bar{f}(s) = L[f(t)]$ and $f'(t)$ is continuous. Also write down the formula for L.T of the n^{th} derivative of f(t). 10

(b) Find inverse L.T of

(i) $\log\left(\frac{1+s}{s}\right)$

(ii) $\dfrac{1}{s^3(s^2+1)}$. 5,5

6. (a) Solve by L.T method

$$\frac{d^2x}{dt^2} + 9x = \cos 2t;$$

if $x(0) = 1$ and $x\left(\frac{\pi}{2}\right) = -1$. 10

(b) An impulsive voltage $E\delta(t)$ is applied to a circuit consisting of L, R, C in series with zero initial conditions. If 'i' be the current at any subsequent time t. Find the limit of 'i' as $t \to 0$. 10

UNIT—IV

7. (a) Form a P.D.E. from

$$z = f(x + at) + g(x - at).$$ 6

(b) Solve

$$y^2p - xyq = x(z - 2y).$$ 6

(c) Solve

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x-y} + \cos(x + 2y).$$ 8

8. (a) Solve

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

by the method of separation of variables. 8

(b) A homogeneous rod of conducting material of length 100 cms has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = x;\ 0 \leq x \leq 50$$
$$= 100 - x;\ 50 \leq x \leq 100.$$

Find the temperature $u(x, t)$ at any time. 12