MATH 2 - JUNE 2005 - 1

Roll No. ..... Total No. of Pages : 3

BT–2/M05 8217

Mathematics–II (2003-04)

Paper : MATH–102E

Time : Three Hours] [Maximum Marks : 100

Note :– Attempt FIVE questions in all, selecting at least ONE question from each part.

PART–A

1. (a) For the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A. 10

(b) When is a square matrix said to be orthogonal ? Verify that the matrix

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$ 10

is orthogonal.

2. (a) Find the values of a and b for which the equations

$x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 3z = 9$

are consistent. When will these equations have a unique solution ? 10

(b) Find the characteristic roots and the corresponding characteristic vectors of the matrix 10

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

PART–B

3. (a) Solve the following equations :

(i) $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

(ii) $(x^2+y^2-a^2)x\,dx+(x^2-y^2-b^2)\,y\,dy=0$ 5+5

(b) Find the orthogonal trajectory of the cardioids

$r=a(1-\cos\theta)$. 10

4. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\,e^x\sin x\,.$$ 10

(b) Solve, by the method of variations of parameters,

$y''-2y'+y=e^x\log x$ 10

5. (a) Solve

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x\,.$$ 10

(b) Solve the simultaneous equations

$$\frac{dx}{dt}+5x-2y=t;\quad \frac{dy}{dt}+2x+y=0$$

given that $x=y=0$ when $t=0$ 10

PART–C

6. (a) Find the linear transform of

(i) $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3$; (ii) $te^{-t}\sin 3t$ 5+5

(b) Find the inverse Laplace forms of

(i) $\frac{s}{(s^2-1)^2}$; (ii) $\frac{4s+5}{(s-1)^2(s+2)}$ 5+5

7. (a) Solve the equation using Laplace transform

$y'' + 4y' + 3y = e^{-t}$, where $y(0) = y'(0) = 1$ 10

(b) (i) Find the Laplace transform of the function

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w; \\ 0, & \pi/w < t < 2\pi/w \end{cases}$$

(ii) Find the inverse Laplace transform of

$$\frac{e^{-cs}}{s^2(s+a)} \quad (c > 0)$$ 5+5

8. (a) Form the partial differential equation by eliminating the arbitrary constants from

$z = a(x+y) + b(x-y) + abt + c$. 8

(b) Solve the partial differential equation

$x^2p^2 + y^2q^2 = z^2$. 6

(c) Solve the partial differential equation using Charpit's method.

$2z + p^2 + qy + 2y^2 = 0$ 6