MATH2-2011-1

BT-2/M-11

MATHEMATICS–II

Paper : MATH-102(E)

(2006 onward)

Time : Three Hours]   [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit.

UNIT–I

1. (a) Find the inverse of the following matrix by Gauss Jordon method :

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$ 10

   (b) Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ 10

2. (a) Verify Cayley-Hamilton theorem for the matrix A and find its inverse :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$ 10

(b) Prove that $\begin{bmatrix} \dfrac{i+1}{2} & \dfrac{i-1}{2} \\ \dfrac{i+1}{2} & \dfrac{1-i}{2} \end{bmatrix}$ is unitary. 10

UNIT–II

3. (a) Solve $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$. 10

   (b) Find the orthogonal trajectories of the family of parabolas $y = ax^2$. 10

4. (a) Solve by the method of variation of parameters :

   $$\frac{d^2y}{dx^2} + y = \cosec x.$$ 10

   (b) A second's pendulum which gains 10 sec/day at one place loses 10 sec/day at another. Compare the acceleration due to gravity at the two places. 10

UNIT–III

5. (a) Evaluate $\displaystyle\int_0^\infty t.e^{-2t} \cos t \, dt$ 5

   (b) Find the inverse Laplace transform of

   $$\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}.$$ 5

   (c) Apply Convolution theorem to evaluate 10

   $$L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}.$$

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6. (a) Solve by Laplace Transform method :

$$y'' + 4y' + 3y = e^{-t} \; ; \; y(0) = y'(0) = 1.$$ 10

   (b) Find the Laplace transform of the function

$$f(t) = \begin{matrix} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi. \end{matrix}$$ 10

UNIT–IV

7. (a) Form a P.D.E. from

$$z = xf_1(x + y) + f_2(x + y).$$ 5

   (b) Solve

$$(y + zx)p - (x + yz)q = x^2 - y^2.$$ 7

   (c) Solve by Charpit's method

$$pxy + pq + qy = yz.$$ 8

8. (a) Solve $r + s - 6t = y \cos x.$ 10

   (b) Using method of separation of variables, solve

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \; ;$$

   given that $u = 3e^{-y} - e^{-5y}$ when $x = 0.$ 10

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