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BT 2/M06 **8252**

Mathematics – II (2005-06)

Paper : MATH-102 E

Time : Three Hours] [Maximum Marks : 100

Note :– Attempt **FIVE** questions in all selecting at least **ONE** question from each unit. Each question carries equal marks.

UNIT–I

1. (i) Reduce the following matrix into its normal form and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ 10

(ii) Find the values of λ for which the equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

are consistent, and find the ratios of x : y : z when λ has the smallest of these values. What happens when λ has the greater of these values ? 10

2. (i) State and prove Cayley-Hamilton Theorem. 10

(ii) If $S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$, where $a = e^{2i\pi/3}$, prove that $S^{-1} = \frac{1}{3}\overline{S}$. 10

UNIT–II

3. (i) Solve : $y\,dx - x\,dy + 3x^2y^2\,e^{x^3}dx = 0$ 10

(ii) Find the Orthogonal Trajectories of the family of confocal conics

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1,$$ λ being the parameter. 10

4. (i) Solve :

$$x^2.\frac{d^2y}{dx^2} + x.\frac{dy}{dx} + y = \log x.\sin(\log x).$$ 10

(ii) A pendulum of length ℓ has one end of string fastened to a peg on a smooth plane inclined at an angle α to the horizon. With the string and the weight on the plane, its time of oscillation is t seconds. If a pendulum of length ℓ' Oscillates in one second when suspended vertically, prove that $\alpha = \sin^{-1}(\ell/\ell' t^2)$. 10

UNIT–III

5. (i) Find the differential equation of all planes which are at a constant distance a from the origin. 10

(ii) Use Charpit's method to solve the partial differential equation : $pxy + pq + qy = yz.$ 10

6. (i) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$, where $u(x, 0) = 6.e^{-3n}$ 10

(ii) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$\begin{aligned} u(x,0) &= x, \quad 0 \le x \le 50 \\ &= 100 - x, \quad 50 \le x \le 100 \end{aligned}$$

Find the temperature u (x, t) at any time. 10

UNIT–IV

7. (i) Prove that

$$\int_0^\infty \frac{e^{-t}.\sin^2 t}{t}dt = \frac{1}{4}\log 5 \qquad 10$$

(ii) Apply convolution theorem to evaluate :

$$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] \qquad 10$$

8. (i) Solve :

$$\frac{d^2x}{dt^2} - t.\frac{dx}{dt} + x = 1,\ \ x(0) = 1,\ \ x'(0) = 2 \qquad 10$$

(ii) A Cantilever beam is clamped at the end $x = 0$ and is free at the end $x = \ell$. It carries a uniform load w per unit length from $x = 0$ to $x = \ell/2$. Calculate the deflection y at any point. 10